

Assignment - of

18/04/24

Q1 Define an Exact Differential Equation

BTL CO PO PSO
1 1 1 -

Sol The given eqn is exact, if $\left[\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \right]$
Working rule:-

Steps ① Compare the given eqn with $Mdx + Ndy = 0$
(Std form) and find M, N , then find

$$\left[\frac{\partial M}{\partial y}, \frac{\partial N}{\partial x} \right]$$

② The given eqn is exact, if $\left[\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \right]$

③ General soln $\int Mdx + \int Ndy = C$
Integrate & eliminate

Q2 Discuss the convergence of the series ~~1 - 1/2! + 1/3! - 1/4! + ...~~ BTL CO PO PSO
2 3 2 -

$$1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \dots$$

Ans This is an alternating series and $u_n = \frac{1}{n!}$

i, clearly $u_n > 0$ and $u_n > u_{n+1} \forall n$
also

$$\text{ii, } \lim_{n \rightarrow \infty} u_n = \lim_{n \rightarrow \infty} \frac{1}{n!} = 0$$

$\therefore$ By Leibnitz's test, $1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} \dots$ series is
convergent.

Q3 Discuss the convergence of the series $\sum \frac{1}{n^2}$

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1	3	12	-

37) given $u_n = \frac{1}{n^2}$

By comparison and Auxiliary test

$$\sum v_n = \frac{1}{n^p}$$

$$\sum v_n = \frac{1}{n^2} \quad \left| \frac{1}{n^p} \right|_{p=2}$$

$\therefore$ If $p > 1$, $\sum v_n$ is convergent

$\therefore$ If $p < 1$, $\sum v_n$ is divergent

If $p = 1$, test fails

$\therefore p > 1$, $\sum v_n$ is convergent

$\therefore \sum u_n$ & $\sum v_n$ behaves the same

$\therefore$ since v_n is convergent then u_n is also convergent.

Q4 Find the orthogonal trajectory of curves $r = c(1 + \cos \theta)$

BT	CO	PO	PSO
2	1	2	1

45) Given $r = c(1 + \cos \theta) \Rightarrow c = \frac{r}{1 + \cos \theta}$

diff 'r' w.r.t θ

$$\frac{dr}{d\theta} = \frac{r(-\sin \theta)}{1 + \cos \theta}$$

$$\frac{dr}{d\theta} = \frac{-r \sin \theta}{1 + \cos \theta}$$

$$\frac{dr}{d\theta} = \frac{-r \sin 2\left(\frac{\theta}{2}\right)}{1 + \cos 2\left(\frac{\theta}{2}\right)}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{-\sin 2(\frac{\theta}{2})}{1 + \cos 2(\frac{\theta}{2})}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \frac{-\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}$$

$$\frac{1}{r} \frac{dr}{d\theta} = -\tan \frac{\theta}{2}$$

$$r \frac{d\theta}{dr} = -\cot \frac{\theta}{2}$$

replace $\frac{r d\theta}{dr}$ by $-\frac{1}{r} \frac{dr}{d\theta}$

$$-\frac{1}{r} \frac{dr}{d\theta} = -\cot \frac{\theta}{2}$$

$$\frac{1}{r} \frac{dr}{d\theta} = \cot \frac{\theta}{2}$$

$$\frac{1}{r} dr = \cot \frac{\theta}{2} d\theta$$

Integrate on both

$$\int \frac{1}{r} dr = \int \cot \frac{\theta}{2} d\theta \quad \therefore \int \cot \frac{\theta}{2} = 2 \log \sin \frac{\theta}{2} + c$$

$$\log r = 2 \log \sin \frac{\theta}{2} + c$$

$$\log r = \log \sin^2 \frac{\theta}{2} + c$$

$$r = \sin^2 \frac{\theta}{2} + c$$

$$r = \frac{c}{2} (1 - \cos \theta), \text{ where } b = \frac{c}{2}$$

$\therefore r = b(1 - \cos \theta)$ is o.t. of the family of curves

⑤ a) solve $x^2 y'' - xy' + y = \log x$

b) solve $(x^3 + 2x^2 + 1)y = x^2 e^{2x} + \sin 2x$

3) solve a) solve $x^2 y'' - xy' + y = \log x$ — ①

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \log x$$

This is Cauchy's Euler equation

Steps

① let $x = e^t$, $t = \log x \rightarrow$ ②

② $x^2 y'' = x^2 y'' = 0(0-1)$
 $x(0) = y' = 0$ } ③

Sub 3, 2 in eq ①

$$0(0-1) - 0 + 1 = \log x \Rightarrow 0^2 - 0 - 0 + 1 = \log x$$

$$\Rightarrow 0^2 - 2\theta + 1 = \log x$$

$F(\theta) y = \log x$

At Eqn. $F(m) = 0$

$$m^2 - 2m + 1 = 0$$

$m = 1, 1$ roots are real and same

$$y_{c.p} = [c_1 + c_2 x] e^x$$

$$y_p = f(\theta) y_p = \log x \quad [\because t = \log x]$$

$$y_p = \frac{\log x}{f(\theta)}$$

$$y_p = \frac{1}{\theta^2 - 2\theta + 1} \log x$$

$$y_p = \frac{1}{\theta^2 - 2\theta + 1} t$$

$\therefore f(\theta) = 1 + \phi(\theta)$

$$\therefore p = [1 + (0^2 - 20)]^{-1} \quad \therefore (1+0)^{-1} = 1 - 0 + 0^2 - 0^3$$

$$y_p = [1 - (0^2 - 2 \cdot 0) + (0^2 - 2 \cdot 0)^2 - (0^2 - 2 \cdot 0)^3 + \dots] t \quad + \quad 0 = 1$$

$$y_p = t + 2 \cdot \quad 0^2 - 0$$

$$y = t + 2$$

$$\rightarrow p = \log x + 2$$

$$\therefore y = y_c + y_p$$

General soln $y = [c_1 + c_2 x]e^x + \log y + 2$

b) Gegeben $(D^3 + 2D^2 + D)y = x^2 e^{2x} + \sin 2x$

Auxiliary form

$$m^3 + 2m^2 + m = 0$$

$m = -1, 0, 0$ roots are real & distinct

$$y_c - f = c_1 e^{-1} x + c_2 e^0 x + c_3 e^0 x$$

$$= c_1 e^{-7t} + c_2 c_1 + c_3 c_1$$

$$= c_1 e^{-x} + c_2 + c_3$$

$$3p = \frac{1}{b^3 + 2b^2 + 0} x^2 e^{2x} \Big|_{b=a+a} + \frac{1}{b^3 + 2b^2 + 0} \sin 2x \Big|_{b^2=a^2}$$

$$g_{22} = \frac{1}{(\cos^2 \theta + 2 \cos \theta + 1)} x^2 + \frac{-1}{\cos^2 \theta + 2 \cos \theta + 1} \sin^2 \theta$$

$$e^{2x} \frac{1}{0^3 + 80^2 + 240 + 18} x^2 + \frac{1}{0(-40) + 2(-40) + 18} \sin 2x$$

$$\frac{1}{18x^2 + 80x + 240} + \frac{1}{-40x + 80} \sin 2x$$

$$e^{2x} \frac{1}{18 \left[1 + \left(\frac{0^2 + 80^2 + 240}{18} \right) \right]} x^2 + \frac{\sin 2x}{-30 - 8}$$

$$\frac{e^{2x}}{18} \left[1 + \left(\frac{0^3 + 60^2 + 210}{18} \right) \right]^{-1} x^2 + \frac{\sin 2x}{-30-8} \times \frac{-30+8}{-30+8}$$

$$\frac{e^{2x}}{18} \left[1 + 0 \right] x^2 + \frac{\sin 2x (-30 + 8)}{90^2 - 64} \Big|_{0^2=4} \quad \boxed{(e+0)^{-1} = 1 - 0 + 0^2 - 0^3}$$

$$\frac{e^{2x}}{18} \left[1 - \frac{0^2 + 8 \cdot 0^2 + 2 \cdot 0}{18} + \frac{0^4 + 16 \cdot 0^3 + 10 \cdot 0^2 + 33 \cdot 0^3 + 4 + 10^2}{324} \right] x^2$$

$$\frac{e^{2x}}{18} \left[1 - \frac{0^3 + 8D^2 + 21D}{28} + \frac{D^4 + 16D^3 + 106D^2 + 336D + 441}{324} \right] x^2$$

$$\frac{e^{2x}}{18} \left(1 - \frac{16+42x}{18} + \frac{882}{324} \right) x^2 + \frac{\sin 2x (-30+8)}{7(4)-64}$$

$$y_p = e^{2x} \left(x^2 - \frac{16+42x}{18} + \frac{882}{324} \right) + \frac{\sin 2x (-30+8)}{-100}$$

$$= e^{2x} \left(x^2 - \frac{16+42x}{18} + \frac{882}{324} \right) + \frac{(-30 \sin 2x + 8 \sin 2x)}{-100}$$

$$= e^{2x} \left(x^2 - \frac{16+42x}{18} + \frac{882}{324} \right) + \frac{(-32 \cos 2x + 8 \sin 2x)}{-100}$$

$$= e^{2x} \left(x^2 - \frac{16+42x}{18} + \frac{882}{324} \right) + \frac{(-6 \cos 2x + 8 \sin 2x)}{-100}$$

$$= e^{2x} \left(x^2 - \frac{16+42x}{18} + \frac{882}{324} \right) + \frac{7x}{100} (3 \cos 2x - 4 \sin 2x)$$

$$y_p = e^{2x} \left(x^2 - \frac{16+42x}{18} + \frac{882}{324} \right) + \left(\frac{1}{50} (3 \cos 2x - 4 \sin 2x) \right)$$

general soln =

$$y = y_c + y_p =$$

$$C_1 e^{-12x} + C_2 + C_3 + e^{2x} \left(x^2 - \frac{16+42x}{18} + \frac{882}{324} \right) + \frac{1}{50} (3 \cos 2x - 4 \sin 2x)$$

60 Test the convergence of the series $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-2)!}$

$$\text{Given } \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(n-2)!}$$

$$\sum u_n = \frac{(-1)^{n-1}}{(n-2)!}$$

$$\sum u_{n+1} = \frac{(-1)^{(n+1)-1}}{(n+1-2)!}$$

$$= \frac{(-1)^n}{(n-1)!}$$

$$= \frac{(-1)^n}{(n)!}$$

$$\frac{u_n}{u_{n+1}} = \frac{(-1)^{n-1}}{(n-2)!} \times \frac{(n)!}{(-1)^n}$$

$$\frac{(-1)^{n-1} \times (n)!}{1} \times \frac{(n)!}{(n-2)!}$$

$$-1 \times (2n) (2n-1)$$

$$-(2n) (2n-1)$$

$$= n \rightarrow \infty - (2n) (2n-1)$$

$$= n \rightarrow \infty - 4n^2 + 2n$$

$$= \lim_{n \rightarrow \infty} -2 \left[\frac{2n^2}{n^2} + \frac{2}{n^2} \right]$$

$$= -\infty$$

$\therefore L < 1$, $\sum u_n$ is convergent

$\therefore$ D'Alembert's test

Solve $\cos^2 x \frac{dy}{dx} + y = 1$

given $\cos^2 x \frac{dy}{dx} + y = 1$

$$\begin{aligned} & \frac{82}{324} x^2 + \frac{\sin 2x (-30 + 8)}{704 - 64} \\ & \frac{82}{324} + \frac{\sin 2x (-30 + 8)}{-100} \\ & \frac{82}{324} + \frac{(-30 \sin 2x + 8 \sin 2x)}{-100} \\ & \frac{82}{324} + \frac{(-30 \cos 2x + 8 \sin 2x)}{-100} \\ & \frac{82}{324} + \frac{(-6 \cos 2x + 8 \sin 2x)}{-100} \\ & \frac{82}{324} + \frac{7x}{100} (3 \cos 2x - 4 \sin 2x) \\ & \frac{82}{324} + \left(\frac{1}{50} (3 \cos 2x - 4 \sin 2x) \right) \end{aligned}$$

$$\left(x^2 \frac{16 + 12x}{18} + \frac{882}{324} \right) \\ (x - 4 \sin x) //$$

$$\text{series} \leq \frac{(-1)^{n+1}}{(n-2)!}$$

$$\frac{(-1)^{n-1} 2^n}{1} \times \frac{2^{n+1} (2n-1) (2n-3)!}{(2n-5)!}$$

$$-1 \times (2n) (2n-1)$$

$$-(2n) (2n-1)$$

$$\lim_{n \rightarrow \infty} -(2n) (2n-1)$$

$$\lim_{n \rightarrow \infty} -4n^2 + 2n$$

$$\lim_{n \rightarrow \infty} -2 \left[2n^2 + \frac{2}{n^2} \right]$$

$$= -\infty$$

$\therefore L < 1$, $\sum u_n$ is divergent

$\therefore$ D'Alembert ratio test $\sum u_n$ is divergent.

Q Solve $\cos^2 x \frac{dy}{dx} + y = \tan x$

$\times$ given $\cos^2 x \frac{dy}{dx} + y = \tan x$

~~divide $\cos^2 x$ on both~~

$$\cos^2 x \left(\frac{dy}{dx} + \frac{y}{\cos^2 x} \right) = \tan x$$

$$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$$

Now std form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\text{Identify } P(x) = \frac{1}{\cos^2 x}; \quad Q(x) = \frac{\tan x}{\cos^2 x}$$

Steps

① Find I.F (Integrating factor)

$$I.F = e^{\int P(x) dx}$$

$$= e^{\int \frac{1}{\cos^2 x} dx}$$

$$= e^{\int \sec^2 x \, dx}$$

$$I.F = e^{\tan x}$$

② Organised soln

$$y \cdot (I.F) = \int (Q(x) \cdot (I.F)) \, dx + C$$

$$\Rightarrow y \cdot e^{\tan x} = \int \frac{\tan x}{\cos^2 x} e^{\tan x} \, dx + C$$

$$\Rightarrow y \cdot e^{\tan x} = \int \tan x \sec^2 x e^{\tan x} \, dx + C$$

$$= \int t \, dt$$

$$t = \tan x$$

$$\frac{dt}{dx} = \sec^2 x$$

$$dt = \sec^2 x \, dx$$

$$\Rightarrow \int t \, dt = \int t \cdot e^t \, dt + C$$

$$= \int t \, dt \quad \text{Acc. to ILATE}$$

$$\boxed{u \cdot v - \int u' \cdot v}$$

$$\Rightarrow \int t \, dt = \int t \cdot e^t - e^t + C$$

$$\text{Sub } t = \tan x$$

$$\Rightarrow y \cdot e^{\tan x} = \int \tan x \cdot e^{\tan x} - e^{\tan x} + C$$

$$\Rightarrow y \cdot e^{\tan x} = e^{\tan x} [\tan x - 1] + C$$

$$= \frac{y \cdot e^{\tan x}}{e^{\tan x}} = \frac{e^{\tan x} [\tan x - 1] + C}{e^{\tan x}}$$

$$= y = \tan x - 1 + C e^{-\tan x}$$

Q2 Obtain the singular solution of

$$y = xy' - \frac{1}{y'}$$

Given $y = xy' - \frac{1}{y'}$ — (1)

Let $y = p$

from eqn (1)

$$y = xp - \frac{1}{p} \quad \text{--- (2)}$$

S-2

diff w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx} \left(xp - \frac{1}{p} \right)$$

$$\frac{dy}{dx} = p$$

$$= \frac{d}{dx} xp - \frac{1}{p^2} \cdot \frac{dp}{dx}$$

$$p = x \frac{dp}{dx} + p - \frac{1}{p^2} \cdot \frac{dp}{dx}$$

$$p = p + x \frac{dp}{dx} - \frac{1}{p^2} \cdot \frac{dp}{dx}$$

$$0 = \left(x + \frac{1}{p^2} \right) \cdot \frac{dp}{dx}$$

Sub p' on b.s

$$0 = \left(x + \frac{1}{p^2} \right) \cdot \frac{dp}{dx}$$

S-3

either $\frac{dp}{dx} = 0$ or $x + \frac{1}{p^2} = 0$

If $\frac{dp}{dx} = 0$ then $p = c$ (const) (leads to general solution)

for the singular solution, consider

$$x + \frac{1}{p^2} = 0$$

$$x = -\frac{1}{p^2}$$

S-4

sub $x = -\frac{1}{p^2}$ into eqn (2)

$$y = \left(-\frac{1}{p^2} \right) p - \frac{1}{p}$$

$$y = -\frac{1}{p} - \frac{1}{p}$$

$$y = -\frac{2}{p}$$

$$p = -2$$

$$p = -\frac{2}{y}$$

S →

$$\text{sub } p = -\frac{2}{y} \text{ into } x = -\frac{1}{p}$$

$$x = -\frac{1}{\left(-\frac{2}{y}\right)}$$

$$x = -\frac{1}{\frac{-2}{y}}$$

$$x = -\frac{y}{-2}$$

$$4x = -y$$

$$y + 4x = 0$$

$$\boxed{y = -4x}$$

Q (a) Define a sequence with an example

A sequence is a set of numbers or algebraic terms that follows a particular pattern or rule to get from one term to the next

$$\text{Ex: } 1 + 3 + 5 + 7 + \dots + 11$$

$$a = 1, d = 2$$

$$= a + (n-1)d$$

$$= 1 + (n-1)2$$

$$= 1 + (2n-2)$$

$$= 2n-1$$

3(b) determine the nature of the series

$$\sum_{n=1}^{\infty} \frac{n! 3^n}{n^n}$$

ty Given $\sum_{n=1}^{\infty} \frac{n! 3^n}{n^n}$

$$\sum u_n = \frac{n! 3^n}{n^n}$$

$$\sum u_{n+1} = \frac{(n+1)! 3^{n+1}}{(n+1)^{n+1}}$$

$$\frac{u_n}{u_{n+1}} = \frac{n! 3^n}{n^n} \times \frac{(n+1)^{n+1}}{(n+1)! 3^{n+1}}$$

$$\frac{u_n}{u_{n+1}} = \frac{n! 3^n}{n^n} \times \frac{(n+1)^{n+1}}{(n+1)! 3^{n+1}}$$

$$(n+1)! = (n+1) \cdot n!$$

$$3^{n+1} = 3^n \cdot 3 = 3^n \cdot 3$$

$$\frac{n! 3^n}{n^n} \times \frac{(n+1)^{n+1}}{(n+1) n! 3^n \cdot 3}$$

$$\frac{1}{n^n} \times \frac{(n+1)^n}{3}$$

$$\frac{(n+1)^n}{3 n^n}$$

S-3

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} \frac{(n+1)^n}{3 n^n} = \lim_{n \rightarrow \infty} \frac{n \left(1 + \frac{1}{n}\right)^n}{3 n^n}$$

$$\lim_{n \rightarrow \infty} \frac{n \left(1 + \frac{1}{n}\right)^n}{3 n^n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{3} \cdot \frac{e}{1} = \frac{1}{3} \cdot \frac{e}{1} = 0.90$$

$\therefore L < 1 \therefore \sum u_n$ is divergent

$\therefore$ By D'Alembert's ratio test,

Solve $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = (1+x)^2$

Given

$$x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = (1+x)^2 \quad \text{--- (1)}$$

$$D = \frac{dy}{dx} \quad D^2 = \frac{d^2 y}{dx^2}$$

$$x^2 D^2 y - 3x D y + 4y = (1+x)^2$$

$$x^2 D^2 y - 3x D y + 4y = (1+x)^2$$

$$(x^2 D^2 - 3x D + 4) y = (1+x)^2$$

Put $x = e^t$

$\log_e x = t$

for $x > 0$

let $D = \frac{d}{dt}$

$x D = 0, x^2 D^2 = D(D-1)$

--- (2)

$$(D(D-1) - 3D + 4) y = 1 + x^2 + 2x$$

$$(D(D-1) - 3D + 4) y = 1 + e^{2t} + 2e^t$$

which is in operator form $f(D) y = Q(t)$

where $f(D) = (D^2 - 4D + 4)$

The Auxiliary form $f(m) = 0$

where

$$m^2 - 4m + 4 = 0$$

$m = 2, 2$; roots are real and same

$$y_{cf} = (C_1 + C_2 x) e^{2x}$$

$t = x$

$t = \log x$

$$y_p = \frac{1}{f(D)} \cdot Q(x)$$

$$= \frac{1}{D^2 - 4D + 4} (1 + e^{2t} + 2e^t)$$

$$= \frac{1}{D^2 - 4D + 4} + \frac{1}{D^2 - 4D + 4} e^{2t} + 2 \frac{1}{D^2 - 4D + 4} e^t$$

$$= \frac{1}{s^2 - 4s + 4} + \frac{1}{s - 2 + 1} e^{2t} + 2 \frac{1}{1 - 2 + 1} e^t$$

$$\text{Since } f(0) = 0$$

$$= \frac{1}{s^2 - 4s + 4} + x \cdot \frac{1}{2s - 4} e^{2t} + 2 e^t$$

$$\begin{aligned} f(0) &= 0^2 - 4 \cdot 0 + 4 \\ f'(0) &= 0 \\ f''(0) &= 2 \cdot 0 - 4 \end{aligned}$$

$$= \frac{1}{s^2 - 4s + 4} + x \cdot \frac{1}{2s - 4} e^{2t} + 2 e^t$$

$$\text{Since } f'(0) = 0$$

$$f''(0) = 2$$

$$= \frac{1}{s^2 - 4s + 4} + x \cdot \frac{x \cdot 1}{2} e^{2t} + 2 e^t$$

$$y_p = \frac{1}{s^2 - 4s + 4} + \frac{x^2 e^{2t}}{2} + 2 e^t$$

$$t = \log x$$

$$y_p = \frac{1}{s^2 - 4s + 4} + \frac{x^2 e^{2 \log x}}{2} + 2 e^{\log x}$$

$$y_p = \frac{1}{s^2 - 4s + 4} + \frac{x^2 e^{\log x^2}}{2} + 2x$$

$$y_p = \frac{1}{s^2 - 4s + 4} + \frac{x^2 \cdot x^2}{2} + 2x$$

$$y_p = \frac{1}{s^2 - 4s + 4} - \frac{x^4}{2} + 2x$$

general soln

$$y = y_c + y_p$$

$$y = (c_1 + c_2) e^{2x} + \frac{1}{s^2 - 4s + 4} + \frac{x^4}{2} + 2x$$

119 Find the orthogonal trajectories of the family of circles passing through $(0, 2)$ and $(0, -2)$

119 given

center lies on y-axis

The general eqn is

$$x^2 + (y-a)^2 = a^2$$

$$x^2 + y^2 - 2axy = 0 \quad \text{--- (1)}$$

Diff eqn (1) w.r.t x

$$2x + 2y \frac{dy}{dx} - 2a \frac{dy}{dx} = 0$$

Now solve $\frac{dy}{dx}$

$$2x + (y-a) \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{y-a} = \frac{-2x}{y-2a} = \frac{-x}{y-a}$$

$$\frac{dy}{dx} = \frac{-x}{y-a} \quad \text{--- (2)}$$

From eqn (1) solve a :-

$$x^2 + y^2 = 2ay$$

$$a = \frac{x^2 + y^2}{2y}$$

sub a in eqn (2)

$$\frac{dy}{dx} = \frac{-x}{y - \frac{x^2 + y^2}{2y}} = \frac{-x}{\frac{2y^2 - x^2 - y^2}{2y}} = \frac{-2xy}{y^2 - x^2}$$

$$\frac{dy}{dx} = \frac{-2xy}{y^2 - x^2} \quad \text{--- (3)}$$

Now we find orthogonal trajectory
so we use the negative reciprocal of the slope

of the family
(0,2) and (0,-2)

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Now use variable separable method

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \Rightarrow 2xy dy = (y^2 - x^2) dx$$

$$\frac{dy}{dx} = 2xy dy - (y^2 - x^2) dx = 0$$

$$\text{Let } v = \frac{y}{x} \Rightarrow y = vx$$

$$\boxed{\frac{dy}{dx} = v + x \frac{dv}{dx}}$$

$$v + x \frac{dv}{dx} = \frac{(v^2 x^2 - x^2)}{2x^2 v} = \frac{x^2(v^2 - 1)}{2x^2 v}$$

$$= \frac{v^2 - 1}{2v}$$

$$v + x \frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$x \frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$x \frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v} = \frac{-v^2 - 1}{2v}$$

$$x \frac{dv}{dx} = \frac{-v^2 - 1}{2v}$$

$$\frac{2v}{-(v^2 + 1)} dv = \frac{1}{x} dx$$

's' on B.S

$$-\int \frac{2v}{(v^2 + 1)} dv = \int \frac{1}{x} dx$$

$$t = v^2 \quad \left| \quad dt = 2v dv \right.$$

$$\frac{dt}{dv} = 2v$$

$$-\int \frac{1}{t+1} dt = \int \frac{1}{x} dx$$

$$-\log|t+1| = \log x$$

$$-\log|v^2 + 1| = \log x + c$$

$$\frac{1}{\sqrt{x+1}} = x+c$$

$$\frac{1}{\frac{x^2}{x^2+1}} = x+c$$

$$\frac{1}{\frac{x^2+x^2}{x^2}} = x+c$$

$$\frac{x^2}{x^2+x^2} = x+c$$

120) Examine the convergence or divergence of the following series:

$$\sum \frac{x^{n+1}}{(n+1)5^n}$$

121) Given $\sum_{n=1}^{\infty} \frac{x^{n+1}}{(n+1)5^n}$

$$\sum u_n = \frac{x^{n+1}}{(n+1)5^n}$$

$$\sum u_{n+1} = \frac{x^{n+2}}{(n+2)5^{n+1}}$$

$$\frac{u_n}{u_{n+1}} = \frac{x^{n+1}}{(n+1)5^n} \times \frac{(n+2)5^{n+1}}{x^{n+2}}$$

$$= \frac{x^n \cdot x}{(n+1)5^n} \times \frac{(n+2)5^{n+1}}{x^n \cdot x}$$

$$\lim_{n \rightarrow \infty} \frac{n+2}{x(n+1)5}$$

$$\frac{1}{x} \cdot 1 \cdot 1$$

IF $L < 1$ i.e. $x < 1$ the series convergent
 IF $L > 1$ i.e. $x > 1$ the series is divergent
 IF $L = 1$ i.e. $x = 1$ test fails.

if $x = 1$ the series becomes $\sum_{n=1}^{\infty} \frac{1}{(n+1)5^n}$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad \left| \frac{1}{n^p} \right| \quad \left| p = \frac{3}{2} > 1 \right|$$

Since $\frac{1}{(n+1)5^n} < \frac{1}{n^{3/2}}$ the series

converges by comparison test.

for $x = -1$

The series becomes $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)5^n}$

This is an alternating series

Since $\frac{1}{(n+1)5^n}$ is decreases and

approaches as $n \rightarrow \infty$, the series converges

The series converges for

$$-1 \leq x \leq 1$$

The series converges in interval $[-1, 1]$

solve $(hx+by+f)dx + (ax+hy+g)dy = 0$.

Given $(hx+by+f)dx + (ax+hy+g)dy = 0$

Steps ① compare the given eqn with form
 $Mdx + Ndy = 0$

$$M = hx+by+f \quad N = ax+hy+g$$

$$\frac{\partial M}{\partial y} = b$$

$$\frac{\partial N}{\partial x} = a$$

S-2 $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow b = a$ its exact

5-3) $\int M dx + \int N dy = c$
 - const - undetermined

$$\int h x^2 + b x + f dx + \int a x + n y + g dy = c$$

$$\frac{h}{2} x^3 + b x^2 + f x + \frac{a}{2} x^2 + n y + g y = c$$

$$= \frac{h x^3}{2} + b x^2 + f x + \frac{a x^2}{2} + n y + g y = c$$

is required soln

14.10 solve $x^2 y'' - 2x y' - 4y = 0$

14.11 Given $x^2 y'' - 2x y' - 4y = 0$ - (1)

$$(x^2 n) y - (2x^2 n) y - 4y = 0$$

By Cauchy's - Euler eqns

$$\left. \begin{aligned} x^2 y'' &= n(n-1) \\ 2xy' &= 0 \end{aligned} \right\} \text{ (2)}$$

$$x = e^t, \quad t = \log x \quad \text{--- (3)}$$

sub (2) & (3) in (1)

$$[n(n-1) - 2n - 4] = 0$$

$$n^2 - 3n - 4 = 0$$

$$n^2 - 4n + n - 4 = 0$$

$$f(n) = 0$$

$$A.E.M. \rightarrow f(n) = 0$$

$$m^2 - 3m - 4 = 0$$

$m = 4, -1$ real & distinct.

$$y_{c.f} = c_1 e^{4x} + c_2 e^{-x}$$

$$y_{c.f} = c_1 e^{4x} + c_2 e^{-x}$$

$$y_p = \frac{1}{f(n)}$$

$$y_p = \frac{1}{n^2 - 3n - 4}$$

$$y = y_c + y_p$$

$$y = c_1 e^{4x} + c_2 e^{-x} + \frac{1}{n^2 - 3n - 4}$$

150 Discuss the convergence of the series

$$1 + \frac{x^2}{2} + \frac{x^2}{8} + \frac{x^2}{10} + \dots \text{ (Queso)}$$

158 Given $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots$

$$\sum_{n=0}^{\infty} \frac{x^n}{n^2+1}$$

$$u_n = n^2 + 1$$

$$0^2 + 1 = 1$$

$$1^2 + 1 = 2$$

$$2^2 + 1 = 5$$

$$3^2 + 1 = 10$$

so

$$\sum_{n=0}^{\infty} \frac{x^n}{n^2+1}$$

$$\sum u_n = \frac{x^n}{n^2+1}$$

$$\sum u_{n+1} = \frac{x^{n+1}}{(n+1)^2+1}$$

$$\frac{u_n}{u_{n+1}} = \frac{x^n}{n^2+1} \times \frac{(n+1)^2+1}{x^{n+1}}$$

$$= \frac{x^n}{n^2+1} \times \frac{(n+1)^2+1}{x^n - x}$$

$$\frac{u_n}{u_{n+1}} = \frac{(n+1)^2+1}{n^2+1}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = \lim_{n \rightarrow \infty} \frac{(n+1)^2+1}{n^2+1} = \lim_{n \rightarrow \infty} \frac{n^2+2n+2}{n^2+1}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 \left[1 + \frac{2}{n} + \frac{2}{n^2} \right]}{n^2 \left[1 + \frac{1}{n^2} \right]}$$

$$1 + \frac{2}{\frac{1}{0}} + \frac{2}{\left(\frac{1}{0}\right)^2}$$

$$\frac{1+0+0}{(1+0)} = 1$$

$$+ 2e^{-x} + \frac{1}{0^2+1}$$

from ratio test

$x < 1$; $L < 1$ converges

$x > 1$; $L > 1$ diverges

$x = 1$; $L = 1$ test fails

5-3

$x = 1$

$$\sum_{n=0}^{\infty} \frac{1}{n^2+1}$$

$$\frac{1}{n^2+1} < \frac{1}{n^2} \text{ and } \sum \frac{1}{n^2} \text{ converges (p=2)}$$

from comparison test $x = 1$ is
converges

The series

$$\sum_{n=0}^{\infty} \frac{x^n}{n^2+1}$$

$\therefore$ converges for $0 \leq x \leq 1$

$\therefore$ diverges for $x > 1$

Solve

16a $(x^3 + y^3)dx - xy^2dy = 0$

16b $(x^3 + y^3)dx - xy^2dy = 0 \quad \text{--- (1)}$

$$M = x^3 + y^3 \quad N = -xy^2$$

$$\frac{\partial M}{\partial y} = 3y^2 \quad \frac{\partial N}{\partial x} = -y^2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad \text{non-exact}$$

I.F for Homogeneous

$$I.F = \frac{1}{Mx + Ny} = \frac{1}{(x^3 + y^3)x + y(-xy^2)}$$

$$= \frac{1}{x^4 + xy^3 - xy^3} = \frac{1}{x^4}$$

Q1. I.F

$$\left(\frac{x^4}{x^4} + \frac{y^3}{x^4}\right) dx - \frac{x^2 y^3}{x^4} dy = 0$$

$$M = \left(\frac{1}{x} + \frac{y^3}{x^4}\right) \quad N = -\frac{y^3}{x^3}$$

soln is $\int M dx + \int N dy = c$
 y -const x -const

$$\int \frac{1}{x} + \frac{y^3}{x^4} dx + \int -\frac{y^3}{x^3} dy = c$$

$$y^3 \int \frac{1}{x} + \frac{1}{x^4} dx + \int -y^2 dy = c$$

$$y^3 \left(\log x + -\frac{1}{3x^3} \right) - \frac{y^3}{3} = c$$

$$\log x \cdot y^3 - \frac{y^3}{3x^3} - \frac{y^3}{3} = c$$

$$\frac{dy}{dx} + 5 \frac{dy}{dx} + 4y = 3e^{-x} + 2x + \sin x$$

Q2 Given

$$\frac{dy}{dx} + 5 \frac{dy}{dx} + 4y = 3e^{-x} + 2x + \sin x$$

$$D^2 y + 5Dy + 4y = 3e^{-x} + 2x + \sin x$$

$$(D^2 + 5D + 4)y = 3e^{-x} + 2x + \sin x$$

Auxiliary form

$$m^2 + 5m + 4 = 0$$

$m = -1, -4$ roots are distinct

$$y_{CF} = C_1 e^{-x} + C_2 e^{-4x}$$

$$P(x) = D^2 + 5D + 4$$

$$y_p = \frac{1}{P(x)} \cdot 3e^{-x} + 2x + \sin x$$

$$y_p = \frac{1}{D^2 + 5D + 4} \cdot 3e^{-x} + \frac{1}{D^2 + 5D + 4} \cdot 2x + \frac{1}{D^2 + 5D + 4} \sin x$$

$$= 3 \frac{1}{1-9+4} e^{-x}$$

since $P(x) \neq 0$

$$3x \frac{1}{20+3} e^{-x} + \frac{1}{0^2+30+4} 2x + \frac{1}{0^2+30+4} \sin x \quad \left| \begin{array}{l} 0.2 \text{ sec} \\ -0.1 \end{array} \right.$$

$$3 \frac{x}{-2+3} e^{-x} + \frac{1}{4(1+\frac{0^2+30}{4})} 2x + \frac{1}{1-3+4} \sin x$$

since $f(0)=0$

$$3 \frac{x}{3} e^{-x} + \frac{2}{4} \left(1 + \frac{0^2+30}{4} \right)^{-1} x + f'(0) = 20+3$$

$$+ \frac{1}{2} (1-0^2+30+0^2+\dots) x + \frac{x}{-2+3} \sin x$$

$$+ \frac{1}{2} \left(1 - \frac{0^2+30}{4} + \frac{(0^2+30)^2}{4} \right) x + \frac{2}{3} \sin x$$

$$+ \frac{1}{2} \left(1 - \frac{9}{4} \right) + \frac{2}{3} \sin x$$

$$+ \frac{1}{2} \left(x - \frac{9}{4} \right) + \frac{2}{3} \sin x$$

$$xe^{-x} + \frac{2}{2} - \frac{9}{8} + \frac{x}{3} \sin x$$

$$y = 2e^{-x} + \frac{2}{2}$$

$$y = (Ce^{-x} + 2e^{-x} + \frac{2}{2} - \frac{9}{8} + \frac{x}{3} \sin x)$$

180 Examine the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{2n+3n}$$

$$\text{S.S. } \sum u_n = \frac{1}{2n+3n}$$

$$\sum u_{n+1} = \frac{1}{2(n+1)+3(n+1)}$$

$$\frac{u_n}{u_{n+1}} = \frac{1}{2n+3n} \times \frac{2(n+1)+3(n+1)}{1}$$

$$= \frac{1}{2n+3n} \times \frac{2n+2+3n+3}{1}$$

$$\Rightarrow 1 \times \frac{5}{1} = 5$$

If $2 \leq \sum u_n$ is convergent

Since D'Alembert's ratio test.

$$\frac{1}{D^2 + D + 4} \sin x \quad \left| \begin{array}{l} D^2 + D + 4 \\ -e^{ix} \\ \hline \end{array} \right.$$

$$\frac{1}{D^2 + D + 4} \sin x$$

since $f(0) = 0$

$$f'(0) = 2D + 5$$

$$+ \frac{2}{-2+5} \sin x$$

$$\left(\frac{2+5}{5} \right) x + \frac{2}{5} \sin x$$

$$e^{-x} + \frac{x}{2} - \frac{3}{8} + \frac{2}{5} \sin x$$

the series

Solve $(D^2 + D + 4)y = e^{-2x} \sin x$ using method of variation of parameter.

$$(D^2 + D + 4)y = e^{-2x} \sin x$$

Auxiliary form

$$m^2 + m + 4 = 0$$

$m = -2, -2$ roots are real & same.

$$y_{c.f.} = C_1 e^{-2x} + C_2 x e^{-2x}$$

P.I. using M.V.P

$$y = A e^{-2x} + B x e^{-2x} \quad y = Au + Bv \quad u = e^{-2x}$$

$$v = e^{-2x} x$$

where

$$A = - \int \frac{u \cdot R}{W} dx \quad B = \int \frac{v \cdot R}{W} dx$$

$$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = \begin{vmatrix} e^{-2x} & x e^{-2x} \\ -2e^{-2x} & e^{-2x} - 2x e^{-2x} \end{vmatrix}$$

$$W = e^{-2x} (e^{-2x} - 2x e^{-2x}) - x e^{-2x} (-2e^{-2x})$$

$$= e^{-4x} - 2x e^{-4x} + 2x e^{-4x} = e^{-4x}$$

$$W = e^{-4x}$$

$R = e^{-2x} \sin x$ in denominator $W = 1 \rightarrow$

$$= e^{-2x} \sin x$$

$$A = - \int \frac{e^{-2x} \sin x \cdot x e^{-2x}}{e^{-4x}} dx = - \int x \sin x dx$$

$$A = x \cos x - \sin x$$

$$A = x \cos x - \sin x$$

$$B = \int \frac{e^{-2x} \sin x \cdot e^{-2x}}{e^{-4x}} dx = \int \sin x dx$$

$$= -\cos x$$

$$y_p = A e^{-2x} + B x e^{-2x}$$

$$= (x \cos x - \sin x) e^{-2x} + C \cos x (x e^{-2x})$$

$$y_p = e^{-2x} (x \cos x - \sin x) - x \cos x e^{-2x}$$

$$y_p = e^{-2x} (x \cos x - \sin x - x \cos x)$$

$$y_p = -e^{-2x} \sin x$$

$$= -\sin x e^{-2x}$$

general soln

$$y = y_c + y_p$$

$$y = (C_1 + C_2 x) e^{-2x} - \sin x e^{-2x}$$

190) Discuss the convergence of the series

$$\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \dots$$

191) given $\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \dots$

$$\sum_{n=2}^{\infty} u_n = \frac{(-1)^{n-1}}{\log n}$$

$$\sum_{n=2}^{\infty} u_{n+1} = \frac{(-1)^n}{\log(n+1)}$$

$$\frac{u_n}{u_{n+1}} = \frac{(-1)^{n-1}}{\log n} \times \frac{\log(n+1)}{(-1)^n}$$

$$= \frac{(-1)^{n-1}}{(-1)^n} \times \frac{\log(n+1)}{\log n}$$

$$= (-1)^n \times \frac{\log(n+1)}{\log n}$$

$$= \frac{\log(n+1)}{\log n} = \frac{(n+1)}{n}$$

$$\lim_{n \rightarrow \infty} -\frac{(n+1)}{n}$$

$$\lim_{n \rightarrow \infty} -\frac{n(1+\frac{1}{n})}{n}$$

$$\lim_{n \rightarrow \infty} -1 - \frac{1}{n}$$

$$= -1 - \frac{1}{0}$$

$$= -1 - 0$$

$$= -1$$

$$L < 1$$

$\therefore u_n$ is divergent

$\therefore \sum u_n$ is ^{divergent} convergent

3) Test for the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^{2n}$$

$$u_n = \frac{(n!)^2}{(2n)!}$$

$$u_{n+1} = \frac{(n+1)!}{(2n+2)!} x^{2n+2} = \frac{(n+1)!}{(2n+2)!} x^{2n+2}$$

$$\frac{u_n}{u_{n+1}} = \frac{(n!)^2}{(2n)!} \times \frac{(2n+2)!}{(n+1)! \cdot x^{2n+2}}$$

$$= \frac{(n!)^2}{2n!} \times \frac{(2n+2)(2n+1)(2n)!}{(n+1)! \cdot x^{2n+2}}$$

$$= \frac{n! (2n+2)(2n+1)}{(n+1) x^{2n+2}}$$

$$= \lim_{n \rightarrow \infty} \frac{n! (2n+2)(2n+1)}{(n+1) x^{2n+2}}$$

$$\lim_{n \rightarrow \infty} = \infty$$

3. S 2016) Define absolute and conditional convergent with an example.

ABSOLUTE CONVERGENCE OF A SERIES

If a convergent series whose terms are not all positive remain convergent when all its terms are made positive, then it is called an absolutely convergent series i.e.,

Then series $\sum u_n$ is said to be absolutely convergent if $\sum |u_n|$ is a convergent series.

CONDITIONAL CONVERGENCE OF A SERIES

A series is said to be conditionally convergent if it is convergent but does not converge absolutely.

Example 1

Test whether the following series are absolutely convergent or conditionally convergent.

Ex:-

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

Q. $u_n = \frac{1}{n^2}$

Q. $u_n > u_{n+1}$

$$\text{If, } \lim_{n \rightarrow \infty} u_n = \lim_{n \rightarrow \infty} \frac{1}{n^2} = 0$$

$\sum u_n$ series is converges

$$\textcircled{2} |u_n| = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$

$$u_n = \frac{1}{n^2}$$

$$v_n = \frac{1}{n^p} \quad \left| \frac{1}{n^2} \right| = |p=2|$$

$$p > 1$$

$\sum |u_n|$ is convergent

u_n and $|u_n|$ are convergent

Series is Absolutely convergent.

Test for the convergence of the series

$$\sum_{n=1}^{\infty} [3\sqrt[n]{n^3+1} - n]$$

$$\log \quad 3\sqrt[n]{n^3+1} = 3\sqrt[n]{n^3(1+\frac{1}{n^3})} = n \cdot 3\sqrt{1+\frac{1}{n^3}}$$

using binomial approximation

$$3\sqrt{1+\frac{1}{n^3}} \approx 1 + \frac{1}{3n^3}$$

$$u_n \approx n \left(1 + \frac{1}{3n^3} \right) - n = \frac{1}{3n^2}$$

By comparison Test

$$v_n = \frac{1}{3n^2} \quad \left(\frac{1}{n^p} = |p=2| \right)$$

$\therefore p > 1, \sum v_n$ is convergent

by comparison Test.

$$\lim_{n \rightarrow \infty} \frac{u_n}{v_n} = \lim_{n \rightarrow \infty} \frac{1}{3n^2} \times \frac{n^2}{1} = \frac{1}{3}$$

$\sum u_n$ and $\sum v_n$ behaves the same
since $\sum v_n$ is convergent; Then $\sum u_n$ is also convergent.